RATIOS, RATES, UNIT RATES & PROPORTIONS

NOTES, GUIDED PRACTICE, &
INDEPENDENT PRACTICE

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RATIOS

A Ratio is a comparison of two quantities. Ratios can be written in 3 different ways.

Example: In a class of 20 students, 12 are girls. Write the ratio of the number of girls to total students.

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<th>As a Fraction</th>
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Ratios can be reduced. Write an equivalent ratio to .

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“Guided” Practice

There are 14 red marbles and 20 blue marbles in a bag of marbles.

A. What is the ratio of red to blue marbles in simplest form?

B. What is the ratio of blue marbles to total marbles in simplest form?
RATES

A **Rate** is a ratio comparing two different units.

Example: *A bike travels 54 miles in 4 hours.*

When written as a rate:

You can write *equivalent rates* by multiplying or dividing the numerator and denominator by the same number.

“Guided” Practice

Sierra can make 8 friendship bracelets in 2 hours. Which of the following rates is equivalent to \( \frac{8 \text{ bracelets}}{2 \text{ hours}} \)?

A. \( \frac{6 \text{ bracelets}}{3 \text{ hours}} \)  
B. \( \frac{3 \text{ bracelets}}{1 \text{ hour}} \)  
C. \( \frac{16 \text{ bracelets}}{4 \text{ hours}} \)  
D. \( \frac{10 \text{ bracelets}}{5 \text{ hours}} \)

Mark can type 44 words in 2 minutes. Which of the following is equivalent to that rate?

A. 62 words in 3 minutes  
B. 84 words in 4 minutes  
C. 100 words in 5 minutes  
D. 132 words in 6 minutes
UNIT RATES

A **Unit Rate** is a ratio of two different measurements where the 2\textsuperscript{nd} measurement is 1.

In “Kid Language” it’s the amount (cost, miles, # of objects, etc) for 1 (1 item, 1 mile, 1 hour, 1 ounce, etc).

Example: **Rate of 180 miles in 3 hours** = \[
\frac{\text{miles}}{\text{hours}}
\]

To turn this into a unit rate, we need the denominator to be 1. In other words, we need to find out how many miles were traveled in 1 hour.

Let’s divide. \[
180 \div 3
\]

Written as a unit rate = \[
\frac{\text{miles}}{\text{hour}}
\]

We can use **Unit Rates** when shopping to help us determine the **better deal**.

Example: You can buy a 40 oz jar of peanut butter for $5.25, or you can buy a 15 oz jar for $2.10. Which is the better deal?

If we find out the cost per ounce for each jar, then we can determine which is the better price. To do this, we need to divide. ALWAYS put “money on top.”

Since the 40 oz. jar is ______ per ounce, and the 15 oz. jar is ______ per ounce, the ______ oz. jar is the better deal. You get more for your money.
“Guided” Practice

A car travels 420 miles on 15 gallons of gas. Express this as a unit rate. (How far does it travel on one gallon of gas?)

At one store, you can purchase 7 apples for $5.46. At another store, you can get 5 apples $3.95. Which is the better deal?

One online music site is advertising 15 song downloads for $6.75. Another site is advertising 12 song downloads for $5.76. Which is the better deal?

PROPORTIONS

A Proportion is an equation stating that two ratios (fractions) are equal.

We can tell if two ratios are equal if their cross products are equal.

For example:

\[ \frac{4}{5} = \frac{8}{10} \]

Example: The average classroom at one school has two boys for every girl. If there are 12 boys in a room, how many girls would there be?

One way to figure it out is to use a drawing.
But that strategy isn’t always reasonable. Would you use a drawing if it asked how many girls would there be if there are 56 boys? No WAY! That’s too much drawing.

We can set up a proportion to solve.

\[
\frac{2 \text{ boys}}{1 \text{ girl}} = \underline{\text{_______}}
\]

Now, since cross products are equal in a proportion, let’s cross multiply.

\[
\frac{2 \text{ boys}}{1 \text{ girl}} = \frac{12 \text{ boys}}{x}
\]

So, there are _____ girls in the class.

“Guided” Practice

Paul reads 5 pages in 4 minutes. How many pages can he read in 7 minutes?

15 dance tickets were sold in one half hour. If this rate continues, how long would you expect it to take to sell 270 tickets?
“On Your Own”

1) Three boxes of baseball cards weigh 90 pounds. How many pounds do 4 boxes weigh?

2) There are 12 cats and 20 dogs in a kennel. What is the ratio of cats to total pets?

3) The ratio of chaperones to students on a field trip to New York City is 2 to 11. If there are 297 students on the trip, how many chaperones are there?

4) A store sells a 28 oz. bottle of shampoo for $1.96. It also sells a 22 oz. bottle of shampoo for $1.32. Which bottle is the better deal?

5) A survey found that 3 out of 5 seventh graders have an email account. If there are 315 seventh graders, how many would you expect to have an email account?
6) A muffin recipe calls for 2 cups of flour and 3 oz. of milk. If the recipe is increased to use 9 cups of flour, how much milk is needed?

7) A bag of candy that contains 30 pieces sells for $4.20. Another bag with 24 pieces sells for $3.60. Which bag is the better deal?

8) A bakery sells 1 sugar cookie for every 4 chocolate chip cookies. How many sugar and chocolate chip cookies could be what they sell?
   A. 2 sugar and 6 chocolate chip
   B. 3 sugar and 10 chocolate chip
   C. 4 sugar and 12 chocolate chip
   D. 5 sugar and 20 chocolate chip

9) A 5 piece pack of gum costs $0.85. An 8 piece pack of gum costs $1.20. How much cheaper per piece is the 8 piece pack?

10) At a light bulb factory, 4 out of every 25 light bulbs are defective. How many light bulbs would you expect to be defective out of 350 light bulbs?
**RATIOS**

A **Ratio** is a comparison of two quantities. Ratios can be written in 3 different ways.

Example: *In a class of 20 students, 12 are girls. Write the ratio of the number of girls to total students.*

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<td>( \frac{12}{20} )</td>
<td>12:20</td>
<td>12 to 20</td>
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Ratios can be reduced. Write an *equivalent ratio* to \( \frac{12}{20} \).

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<td>( \frac{3}{5} )</td>
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“Guided” Practice

There are 14 red marbles and 20 blue marbles in a bag of marbles.

**A.** What is the ratio of red to blue marbles in simplest form?

\[
\frac{14 \text{ red}}{20 \text{ blue}} = \frac{7}{10}
\]

**B.** What is the ratio of blue marbles to total marbles in simplest form?

\[
\frac{20 \text{ blue}}{34 \text{ total}} = \frac{10}{17}
\]
RATES

A Rate is a ratio comparing two different units.

Example: A bike travels 54 miles in 4 hours.

When written as a rate: \( \frac{54 \text{ miles}}{4 \text{ hours}} \)

You can write equivalent rates by multiplying or dividing the numerator and denominator by the same number.

\[
\frac{54 \text{ miles}}{4 \text{ hours}} \div 2 = \frac{27 \text{ miles}}{2 \text{ hours}} \quad \frac{54 \text{ miles}}{4 \text{ hours}} \times 3 = \frac{162 \text{ miles}}{12 \text{ hours}}
\]

“Guided” Practice

Sierra can make 8 friendship bracelets in 2 hours. Which of the following rates is equivalent to \( \frac{8 \text{ bracelets}}{2 \text{ hours}} \)?

B. \( \frac{6 \text{ bracelets}}{3 \text{ hours}} \)  B. \( \frac{3 \text{ bracelets}}{1 \text{ hour}} \)  C. \( \frac{16 \text{ bracelets}}{4 \text{ hours}} \)  D. \( \frac{10 \text{ bracelets}}{5 \text{ hours}} \)

Mark can type 44 words in 2 minutes. Which of the following is equivalent to that rate?

E. 62 words in 3 minutes  
F. 84 words in 4 minutes  
G. 100 words in 5 minutes  
H. 132 words in 6 minutes
UNIT RATES

A Unit Rate is a ratio of two different measurements where the 2\textsuperscript{nd} measurement is 1.

In “Kid Language” it’s the amount (cost, miles, # of objects, etc) for 1 (1 item, 1 mile, 1 hour, 1 ounce, etc).

Example: \textit{Rate of 180 miles in 3 hours} = \frac{180 \text{ miles}}{3 \text{ hours}}

To turn this into a unit rate, we need the denominator to be 1. In other words, we need to find out how many miles were traveled in 1 hour.

Let’s divide. \[ 180 \div 3 = \frac{60}{3} \]

Written as a unit rate = \frac{60 \text{ miles}}{1 \text{ hour}}

We can use Unit Rates when shopping to help us determine the better deal.

Example: You can buy a 40 oz jar of peanut butter for $5.25, or you can buy a 15 oz jar for $2.10. Which is the better deal?

If we find out the cost per ounce for each jar, then we can determine which is the better price. To do this, we need to divide. ALWAYS put “money on top.”

\[
\begin{align*}
\text{40 oz} & : \frac{5.25}{40} = 0.13125 = 0.13 \\
\text{15 oz} & : \frac{2.10}{15} = 0.14
\end{align*}
\]

Since the 40 oz. jar is $0.13 per ounce, and the 15 oz. jar is $0.14 per ounce, the 40 oz. jar is the better deal. You get more for your money.
“Guided” Practice

A car travels 420 miles on 15 gallons of gas. Express this as a unit rate. (How far does it travel on one gallon of gas?)

\[
\frac{420 \text{ miles}}{15 \text{ gallons}} = 420 \div 15 = \frac{28 \text{ miles}}{1 \text{ gallon}}
\]

At one store, you can purchase 7 apples for $5.46. At another store, you can get 5 apples $3.95. Which is the better deal?

\[
\frac{\$5.46}{7 \text{ apples}} = \frac{\$0.78}{1 \text{ apple}} \quad \text{and} \quad \frac{\$3.95}{5 \text{ apples}} = \frac{\$0.79}{1 \text{ apple}}
\]

So, 7 apples for $5.46 is the better deal.

One online music site is advertising 15 song downloads for $6.75. Another site is advertising 12 song downloads for $5.76. Which is the better deal?

\[
\frac{\$6.75}{15 \text{ downloads}} = \frac{\$0.45}{1 \text{ download}} \quad \text{and} \quad \frac{\$5.76}{12 \text{ downloads}} = \frac{\$0.48}{1 \text{ download}}
\]

So, 15 downloads for $6.75 is the better deal.

PROPORTIONS

A **Proportion** is an equation stating that two ratios (fractions) are equal.

We can tell if two ratios are equal if their cross products are equal.

For example:

\[
\frac{40}{5} = \frac{8}{10}
\]

Example: *The average classroom at one school has two boys for every girl. If there are 12 boys in a room, how many girls would there be?*

One way to figure it out is to use a drawing.
But that strategy isn’t always reasonable. Would you use a drawing if it asked how many girls would there be if there are 56 boys? No WAY! That’s too much drawing.

We can set up a proportion to solve.

\[
\frac{2 \text{ boys}}{1 \text{ girl}} = \frac{12 \text{ boys}}{x}
\]

Now, since cross products are equal in a proportion, let’s cross multiply:

\[
\frac{2 \text{ boys}}{1 \text{ girl}} \times x = \frac{12 \text{ boys}}{x} \times 1 \text{ girl}
\]

\[
12 = 2x
\]

\[
\frac{12}{2} = \frac{2x}{2}
\]

\[
6 = x
\]

So, there are 6 girls in the class.

“Guided” Practice

Paul reads 5 pages in 4 minutes. How many pages can he read in 7 minutes?

\[
\frac{5 \text{ pages}}{4 \text{ min}} = \frac{x}{7 \text{ min}}
\]

8.75 pages

15 dance tickets were sold in one half hour. If this rate continues, how long would you expect it to take to sell 270 tickets?

\[
\frac{15 \text{ tickets}}{0.5 \text{ hrs}} = \frac{270 \text{ tickets}}{x}
\]

9 hours
"On Your Own"

1) Three boxes of baseball cards weigh 90 pounds. How many pounds does 4 boxes weigh?

\[
\frac{3 \text{ boxes}}{90 \text{ pounds}} = \frac{4 \text{ boxes}}{x}
\]

\[
x = 120 \text{ pounds}
\]

2) There are 12 cats and 20 dogs in a kennel. What is the ratio of cats to total pets?

\[
\frac{12 \text{ cats}}{32 \text{ total}} = \frac{3}{8}
\]

3) The ratio of chaperones to students on a field trip to New York City is 2 to 11. If there are 297 students on the trip, how many chaperones are there?

\[
\frac{2 \text{ chaperones}}{11 \text{ students}} = \frac{x}{297 \text{ students}}
\]

\[
x = 54 \text{ chaperones}
\]

4) A store sells a 28 oz. bottle of shampoo for $1.96. It also sells a 22 oz. bottle of shampoo for $1.32. Which bottle is the better deal?

\[
\frac{28 \text{ oz bottle}}{1 \text{ oz}} = \frac{0.07}{1 \text{ oz}} \quad \frac{22 \text{ oz bottle}}{1 \text{ oz}} = \frac{0.06}{1 \text{ oz}}
\]

The 22 oz. bottle is the better deal.

5) A survey found that 3 out of 5 seventh graders have an email account. If there are 315 seventh graders, how many would you expect to have an email account?

\[
\frac{3 \text{ have}}{5 \text{ total}} = \frac{x}{315 \text{ total}} \quad x = 189 \text{ have an email account}
\]
6) A muffin recipe calls for 2 cups of flour and 3 oz. of milk. If the recipe is increased to use 9 cups of flour, how much milk is needed?

\[
\frac{2 \text{ Cups Flour}}{3 \text{ oz. Milk}} = \frac{9 \text{ Cups Flour}}{x} \quad x = 13.5 \text{ oz milk}
\]

7) A bag of candy that contains 30 pieces sells for $4.20. Another bag with 24 pieces sells for $3.60. Which bag is the better deal?

30 piece bag: \(\frac{$4.20}{30\text{ pieces}} = \frac{$0.14}{1\text{ piece}}\)

24 piece bag: \(\frac{$3.60}{24\text{ pieces}} = \frac{$0.15}{1\text{ piece}}\)

The 30 piece bag is the better deal.

8) A bakery sells 1 sugar cookie for every 4 chocolate chip cookies. How many sugar and chocolate chip cookies could be what they sell?

E. 2 sugar and 6 chocolate chip
F. 3 sugar and 10 chocolate chip
G. 4 sugar and 12 chocolate chip
H. 5 sugar and 20 chocolate chip

9) A 5 piece pack of gum costs $0.85. An 8 piece pack of gum costs $1.20. How much cheaper per piece is the 8 piece pack.

\[
\frac{$0.85}{5\text{ pieces}} = \frac{$0.17}{1\text{ piece}} \quad \frac{$1.20}{8\text{ pieces}} = \frac{$0.15}{1\text{ piece}}
\]

It’s $0.02 cheaper per piece in the 8 piece pack.

10) At a light bulb factory, 4 out of every 25 light bulbs are defective. How many light bulbs would you expect to be defective out of 350 light bulbs?

\[
\frac{4 \text{ broken}}{25 \text{ total bulbs}} = \frac{x}{350 \text{ total bulbs}} \quad x = 56 \text{ defective bulbs}
\]